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**REDUCING THE ERROR ON  $\alpha$  in  $B \rightarrow \pi\pi, \rho\rho, \rho\pi$** 

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Theoretical errors in the extraction of  $\alpha$  from  $B \rightarrow (\pi^+\pi^-, \rho^+\rho^-, \rho^\pm\pi^\mp)$  decays are usually given in terms of upper bounds on  $|\alpha_{\text{eff}} - \alpha|$  obtained from isospin or from SU(3) relations, where  $\alpha_{\text{eff}}$  is measured through CP asymmetries. We show that mild assumptions about magnitudes and strong phases of penguin and tree amplitudes ( $|P/T| \leq 1$  and  $|\delta| \leq \pi/2$ ) in  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$ , imply  $\alpha_{\text{eff}} > \alpha$ , thus reducing by a factor two the error in  $\alpha$ . Similarly, the assumptions  $|p_\pm/t_\pm| \leq 1$ ,  $|\delta_-| \leq \pi/2 \leq |\delta_+|$  in  $B \rightarrow \rho\pi$  lead to a cancellation between two terms in  $\alpha_{\text{eff}} - \alpha$ . Current data support these conditions, which are justified by both QCD-factorization and flavor SU(3).

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Direct extraction of the Cabibbo-Kobayashi-Maskawa (CKM) phase  $\alpha \equiv \phi_2$  from the time-dependent CP asymmetry in  $B^0 \rightarrow \pi^+\pi^-$  is obstructed by the penguin amplitude [1]. This obstacle may be overcome using isospin symmetry [2], which incorporates electroweak penguin contributions at a percent level [3] but does not include small isospin breaking effects [4]. This method requires separate rate measurements of  $B^0, \bar{B}^0 \rightarrow \pi^0\pi^0$ . As long as low statistics does not permit these separate measurements, one can use combined  $B$  and  $\bar{B}$  decay rates into the three  $\pi\pi$  channels to obtain upper bounds on  $|\alpha_{\text{eff}}^{\pi\pi} - \alpha|$  [5, 6, 7]. The angle  $\alpha_{\text{eff}}^{\pi\pi}$ , which equals  $\alpha$  in the limit of a vanishing penguin amplitude, is given up to a discrete ambiguity by the time-dependent CP asymmetry in  $B^0 \rightarrow \pi^+\pi^-$ . The upper bound on  $|\alpha_{\text{eff}}^{\pi\pi} - \alpha|$ , improved by information from an upper limit on the asymmetry in  $B \rightarrow \pi^0\pi^0$ , remains an intrinsic theoretical uncertainty in  $\alpha$ . Note that  $\alpha_{\text{eff}}^{\pi\pi} - \alpha$  may be either positive or negative.

Similar considerations can also be applied to  $B^0 \rightarrow \rho^+\rho^-$  and lead to analogous bounds on  $|\alpha_{\text{eff}}^{\rho\rho} - \alpha|$ . Because each of the two  $\rho$  mesons carries a unit spin, one must distinguish between decays to even-CP and odd-CP final states corresponding to definite polarizations [8].

A complete isospin analysis of the processes  $B \rightarrow \rho\pi$  is complicated by the existence of five different  $\rho\pi$  charge states in  $B^0$  and  $B^+$  decays [9]. Thus, it was proposed

to measure  $\alpha$  through the time-dependent Dalitz distribution in  $B^0 \rightarrow \pi^+\pi^-\pi^0$  [10], which provides information about interference of amplitudes for  $B^0 \rightarrow \rho^+\pi^-$ ,  $B^0 \rightarrow \rho^-\pi^+$  and  $B^0 \rightarrow \rho^0\pi^0$ . Alternatively, one may measure an angle  $\alpha_{\text{eff}}^{\rho\pi}$  in quasi two-body decays  $B^0(t) \rightarrow \rho^\pm\pi^\mp$ , and use broken flavor SU(3) to obtain an upper bound on  $|\alpha_{\text{eff}}^{\rho\pi} - \alpha|$  [11].

The experimental progress during the past year has been impressive in this class of measurements. The situation in early July [12] was updated in late August [13], following new measurements reported at the International Conference on High Energy Physics in Beijing [14]. The overall range of  $\alpha$  determined in  $B \rightarrow \pi\pi$ ,  $\rho\rho$ ,  $\rho\pi$  [14],  $\alpha = (100^{+12}_{-11})^\circ$ , overlaps with and begins to be narrower than the following bounds obtained indirectly in an independent global CKM fit [15],

$$78^\circ \leq \alpha \leq 122^\circ, \quad 38^\circ \leq \gamma \leq 80^\circ, \quad (1)$$

where a 95% confidence level (CL) is implied. The error in  $\alpha$  from a time-dependent Dalitz plot analysis of  $B \rightarrow \pi^+\pi^-\pi^0$  [14],  $(^{+27}_{-17} \pm 6)^\circ$ , is statistics-dominated. On the other hand, the theoretical errors in determining  $\alpha$ , using isospin in  $B \rightarrow \pi\pi$ ,  $\rho\rho$  and applying broken SU(3) to  $B \rightarrow \rho\pi$ , are at least as large as the corresponding statistical errors. The 90% CL upper bounds on  $|\alpha_{\text{eff}} - \alpha|$  in these three cases are [11, 14, 16]  $37^\circ$ ,  $15^\circ$  and  $17^\circ$ , respectively. It would be very useful to reduce these intrinsic theoretical uncertainties using present data.

In this letter we point out that the above theoretical errors in  $\alpha$  may be reduced by about a factor two under very mild and reasonable assumptions, which are justified by both QCD-factorization and flavor SU(3). In  $B \rightarrow \pi^+\pi^-$  and  $B \rightarrow \rho^+\rho^-$  we study the dependence of  $\alpha_{\text{eff}} - \alpha$  on the ratio of penguin and tree amplitudes,  $r \equiv |P/T|$ , and on their relative strong phase,  $\delta$ . We show that the conditions  $|P/T| \leq 1$  and  $|\delta| \leq \pi/2$  predict a positive sign for  $\alpha_{\text{eff}} - \alpha$ . In  $B \rightarrow \rho^\pm\pi^\mp$  two small ratios of penguin and tree amplitudes,  $r_\pm$ , and two phases  $\delta_\pm$  lying in opposite hemispheres, tend to suppress  $\alpha_{\text{eff}} - \alpha$  due to a cancellation between two terms. We propose to include, in future studies of  $\alpha$ , the explicit dependence of  $\alpha_{\text{eff}} - \alpha$  on  $r$ ,  $\delta$ ,  $r_\pm$  and  $\delta_\pm$ , together with the dependence of the CP asymmetries on these variables. In the argumentation below we assume  $\gamma < 90^\circ$  as given in (1).

To prove our point, we consider first  $B^0 \rightarrow \pi^+\pi^-$ . We use the  $c$ -convention defined in [17], in which the top-quark has been integrated out in the  $b \rightarrow d$  penguin transition and unitarity of the CKM matrix has been used. Absorbing a  $P_{tu}$  term in  $T$ , the decay amplitude may be written in the following general form,

$$A(B^0 \rightarrow \pi^+\pi^-) = T + P = |T| (e^{i\gamma} + r e^{i\delta}), \quad (2)$$

where by convention  $r > 0$ ,  $-\pi < \delta \leq \pi$ . The phase  $\alpha_{\text{eff}}^{\pi\pi}$  is extracted up to a discrete ambiguity from the two asymmetries  $S_{\pi\pi}$  and  $C_{\pi\pi}$  in  $B^0(t) \rightarrow \pi^+\pi^-$  [18]:

$$\sin(2\alpha_{\text{eff}}^{\pi\pi}) = \frac{S_{\pi\pi}}{\sqrt{1 - C_{\pi\pi}^2}}, \quad (3)$$

where

$$C_{\pi\pi} \equiv \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2} = \frac{2r \sin \delta \sin \gamma}{R_{\pi\pi}}, \quad (4)$$

$$S_{\pi\pi} \equiv \frac{2\text{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2} = \frac{\sin 2\alpha - 2r \cos \delta \sin(\alpha - \beta) - r^2 \sin 2\beta}{R_{\pi\pi}}, \quad (5)$$

$$R_{\pi\pi} \equiv 1 + 2r \cos \delta \cos \gamma + r^2. \quad (6)$$

Using the definitions

$$\alpha_{\text{eff}}^{\pi\pi} \equiv \frac{1}{2} \text{Arg} \lambda_{\pi\pi}, \quad \lambda_{\pi\pi} \equiv e^{-2i\beta} \frac{A(\bar{B}^0 \rightarrow \pi^+\pi^-)}{A(B^0 \rightarrow \pi^+\pi^-)}, \quad (7)$$

one has

$$\alpha_{\text{eff}}^{\pi\pi} - \alpha = \frac{1}{2} \arctan \left[ \frac{2r \sin \gamma (\cos \delta + r \cos \gamma)}{1 - r^2 + 2r \cos \gamma (\cos \delta + r \cos \gamma)} \right]. \quad (8)$$

Two very reasonable assumptions,  $r \leq 1$  and  $\cos \delta > -r \cos \gamma$ , imply  $\alpha_{\text{eff}}^{\pi\pi} - \alpha > 0$ . The second condition may be replaced by a stronger one,  $\cos \delta \geq 0$ ; it is stronger since we are assuming  $\gamma < 90^\circ$ . The result  $\alpha_{\text{eff}}^{\pi\pi} > \alpha$  demonstrates the central point of this note.

Rewriting Eq. (8),

$$\cot 2(\alpha_{\text{eff}}^{\pi\pi} - \alpha) = \cot \gamma + \frac{1 - r^2}{2r \sin \gamma (\cos \delta + r \cos \gamma)}, \quad (9)$$

we see that the above assumptions imply  $\cot 2(\alpha_{\text{eff}}^{\pi\pi} - \alpha) > \cot \gamma$ , or  $0 < \alpha_{\text{eff}}^{\pi\pi} - \alpha < \gamma_{\text{max}}/2 = 40^\circ$ . The prediction of the sign of  $\alpha_{\text{eff}}^{\pi\pi} - \alpha$  is based purely on definitions and on the assumptions on  $r$  and  $\delta$ . It implies a considerably narrower range for  $\alpha$  than the bound  $|\alpha_{\text{eff}}^{\pi\pi} - \alpha| < 37^\circ$  [14, 16], obtained from the  $B \rightarrow \pi\pi$  and  $\bar{B} \rightarrow \pi\pi$  isospin triangles, with the inclusion of some information about the asymmetry in  $B \rightarrow \pi^0\pi^0$ .

It now remains to justify our two assumptions,  $r \leq 1$  and  $|\delta| \leq \pi/2$ . The ratio  $r$  has a very long history, starting in the late eighties when the penguin amplitude in  $B \rightarrow \pi^+\pi^-$  was estimated to be small but non-negligible [1, 19]. First measurements of  $B \rightarrow \pi\pi$  and  $B \rightarrow K\pi$  decay rates, performed several years later by the CLEO collaboration [20], were analyzed within flavor SU(3) indicating that  $r \sim 0.3$  [21]. A recent global SU(3) fit to all  $B \rightarrow \pi\pi$  and  $B \rightarrow K\pi$  decays obtained an unexpected large value [22]  $r = 0.69 \pm 0.09$ . The large value of  $r$  is driven partly but not only [23] by a large input value for  $|C_{\pi\pi}|$  [24, 25, 26]. Theoretical calculations based on QCD and a heavy quark expansion [27, 28] find somewhat smaller values for  $r$ , all lying comfortably in the range  $r < 0.5$ . All these calculations support strongly the assumption  $r \leq 1$ .

Very early theoretical QCD arguments favoring a small value of  $|\delta|$  were proposed in [29]. Although different calculations of  $\delta$  [27, 28], based on QCD and a heavy quark expansion, do not always agree in detail, all these computed values of  $|\delta|$  are considerably smaller than  $90^\circ$ . Constraints on CKM parameters based on the assumption  $|\delta| \leq 90^\circ$  and on a given range for  $S_{\pi\pi}$  were studied in [30]. Long distance  $c\bar{c}$  penguin contributions [31], or equivalently final state rescattering [32, 33], may spoil the QCD calculations [26]. These effects cannot be calculated in a model-independent way. A global SU(3) fit to  $B \rightarrow \pi\pi$  and  $B \rightarrow K\pi$  decays, which effectively includes these rescattering effects while assuming SU(3) invariant strong phases, obtains [22]

$\delta = (-34_{-25}^{+11})^\circ$ . Large SU(3) breaking effects in strong phases could possibly lead to values of  $\delta$  outside the range  $|\delta| \leq \pi/2$ .

A study of the two hadronic parameters,  $r$  and  $\delta$ , was performed in [25], adding measurements of  $S_{\pi\pi}$ ,  $C_{\pi\pi}$  and an upper bound on  $|\alpha_{\text{eff}}^{\pi\pi} - \alpha|$  available before this summer to all other CKM constraints [15]. Values were found in an overall minimum  $\chi^2$  fit,  $r = 0.77_{-0.34}^{+0.58}$  and  $\delta = (-43_{-21}^{+14})^\circ$ , consistent with  $|\delta| \leq 90^\circ$  but possibly violating somewhat the bound  $r \leq 1$ . A recent update [34], using newer measurements, also favors parameters in the range  $r \leq 1$ ,  $-90^\circ \leq \delta \leq 0^\circ$ .

Similarly, we attempt a direct experimental proof of  $r \leq 1$  and  $|\delta| \leq \pi/2$  using current data [14, 35]:

$$S_{\pi\pi} = -0.61 \pm 0.14 \text{ (0.32)} , \quad C_{\pi\pi} = -0.37 \pm 0.11 \text{ (0.27)} , \quad |\alpha_{\text{eff}}^{\pi\pi} - \alpha| < 37^\circ \text{ (90\% CL)} , \quad (10)$$

The world averaged asymmetries are based on most recent measurements by the Belle collaboration [36] ( $S_{\pi\pi} = -1.00 \pm 0.21 \pm 0.07$ ,  $C_{\pi\pi} = -0.58 \pm 0.15 \pm 0.07$ ) and by the BaBar collaboration [37] ( $S_{\pi\pi} = -0.30 \pm 0.17 \pm 0.03$ ,  $C_{\pi\pi} = -0.09 \pm 0.15 \pm 0.04$ ). Since these two measurements are not in good agreement with each other, an error rescaling factor of 2.39 [16] (determining errors in parentheses) may be used to achieve a conservative confidence level. Expressions for  $C_{\pi\pi}$ ,  $S_{\pi\pi}$ , and  $|\alpha_{\text{eff}}^{\pi\pi} - \alpha|$ , given in Eqs. (4)(5) and (8), and the values in Eq. (10) can be used to constrain  $r$ ,  $\delta$  and  $\alpha$ .

We performed a separate minimum- $\chi^2$  fit for the parameters  $r$  and  $\delta$ . The  $\chi^2$  contains two contributions from  $C_{\pi\pi}$  and  $S_{\pi\pi}$  and the usual terms corresponding to the standard analysis of the unitarity triangle. In the minimization we reject points which yield  $|\alpha_{\text{eff}}^{\pi\pi} - \alpha| \geq 37^\circ$ . Fig. 1a shows the resulting 90% CL bounds obtained using the unscaled (dark area) and rescaled (dashed line) errors in Eq. (10). The bounds can be summarized by

$$\begin{aligned} 0.2 \text{ (0.05)} < r < 1 \text{ (1.65)} , \\ -107^\circ \text{ (-140}^\circ) < \delta < -19^\circ \text{ (10}^\circ) , \end{aligned} \quad (11)$$

where the numbers in parentheses correspond to the rescaled errors. Note that the unscaled errors exclude  $r \geq 1$  at 90% CL, favoring  $-90^\circ \leq \delta \leq 0$ . Although they do not exclude completely  $\delta < -90^\circ$ , we find that the lowest allowed value of  $\alpha_{\text{eff}}^{\pi\pi} - \alpha$  is  $-3^\circ$ , quite close to zero. The rescaled errors imply less restrictive bounds. Exclusion of  $r > 1$ ,  $\delta < -90^\circ$  would imply  $\alpha < \alpha_{\text{eff}}^{\pi\pi} = 110.5^\circ$  for the central values given in Eq. (10). In any case, using the dependence of  $\alpha_{\text{eff}}^{\pi\pi} - \alpha$ ,  $S_{\pi\pi}$  and  $C_{\pi\pi}$  on  $r$  and  $\delta$  is expected to reduce the theoretical uncertainty in  $\alpha$  below the upper limit on  $|\alpha_{\text{eff}}^{\pi\pi} - \alpha|$ .

One may study directly the dependence of the sign of  $\alpha_{\text{eff}}^{\pi\pi} - \alpha$  on the values of  $C_{\pi\pi}$  and  $S_{\pi\pi}$ . Assuming that the discrepancy between the BaBar and Belle results disappears and taking errors  $\delta C_{\pi\pi} \sim \delta S_{\pi\pi} \sim 0.1$ , we calculate central values in the  $(C_{\pi\pi}, S_{\pi\pi})$  plane for which  $\alpha_{\text{eff}}^{\pi\pi} - \alpha > 0$  at 90% CL. An important input is our prior knowledge of  $\gamma$ . For this we use three different analyses of the unitarity triangle described in Refs. [25], [15] and [38]. Corresponding regions in the  $(C_{\pi\pi}, S_{\pi\pi})$  plane, *outside* of which  $\alpha_{\text{eff}}^{\pi\pi} - \alpha > 0$ , are described in Fig. 2 by the shaded area and by the dashed and dotted lines, respectively. The heavy dot, denoting the current central

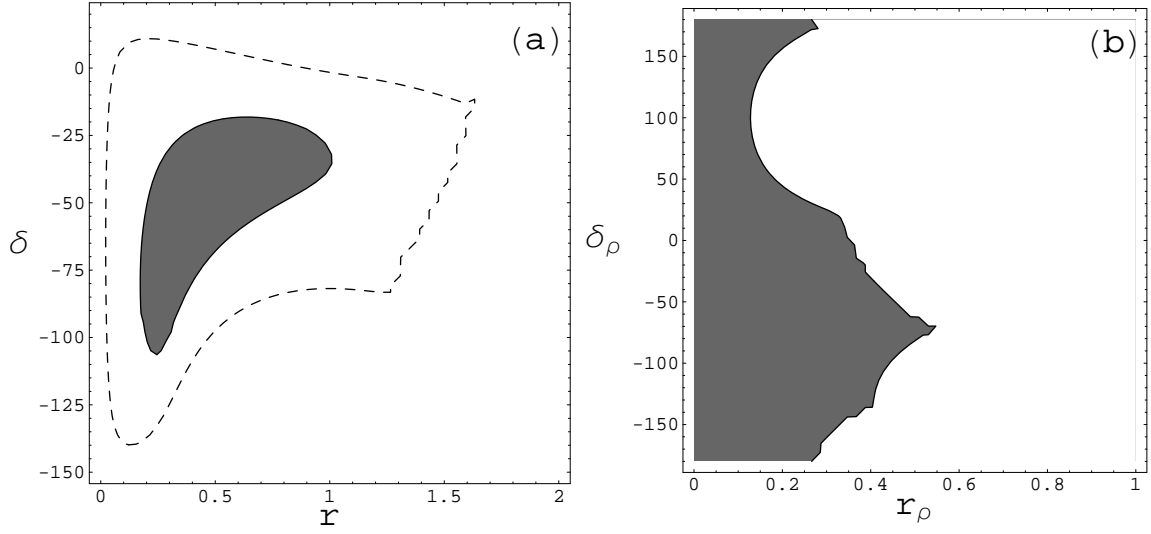


Figure 1: **(a)** Bounds on  $r$  and  $\delta$  at 90% CL based on a  $\chi^2$  analysis of Eq. (10). The shaded and dashed areas correspond to unscaled and rescaled errors in (10), respectively. **(b)** 90% CL bound on  $r_\rho$  and  $\delta_\rho$  based on (12).

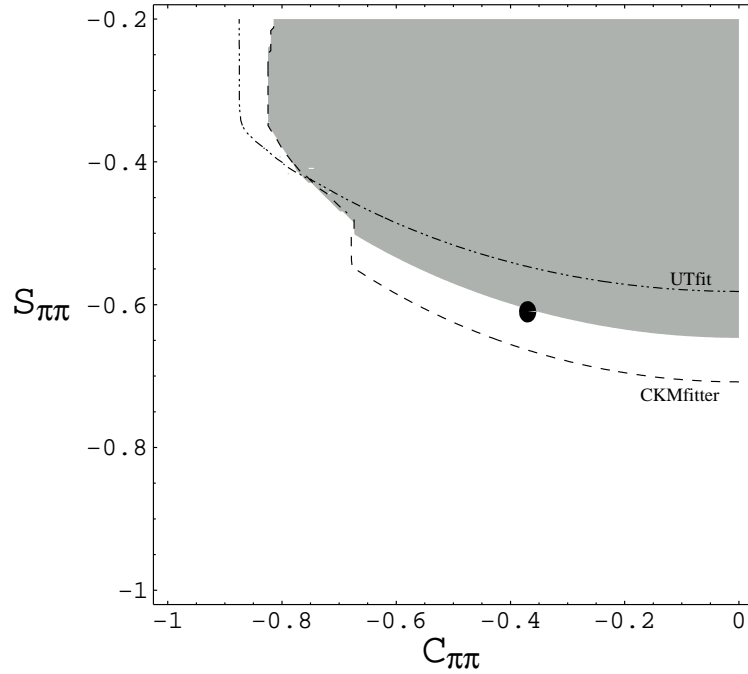


Figure 2: Regions in the  $(C_{\pi\pi}, S_{\pi\pi})$  plane *outside* of which  $\alpha_{\text{eff}}^{\pi\pi} - \alpha > 0$  at 90% CL. The shaded area is obtained using the UT analysis presented in Ref. [25]. The dashed and dotted lines correspond to analyses of the unitarity triangle by the CKMfitter [15] and UTfit [38] collaborations. The heavy dot denotes the current central values (10).

values of  $C_{\pi\pi}$  and  $S_{\pi\pi}$ , is seen to lie close to the bounds but still *inside* the most conservative region [15].

Let us now turn to discuss briefly  $\alpha_{\text{eff}}^{\rho\rho} - \alpha$  in  $B \rightarrow \rho^+\rho^-$ . Using the approximately pure longitudinal polarization of the final states [14, 39], the isospin analysis of the decays  $B \rightarrow \rho\rho$  is greatly simplified, becoming almost identical to that of  $B \rightarrow \pi\pi$  (neglecting the small  $\rho$ -width effect [40]). The small branching ratio for  $B \rightarrow \rho^0\rho^0$  [41] leads to the rather tight 90% CL upper bound [14, 16],  $|\alpha_{\text{eff}}^{\rho\rho} - \alpha| < 15^\circ$ . The phase difference  $\alpha_{\text{eff}}^{\rho\rho} - \alpha$  may be written as in Eq. (8) and becomes positive under similar conditions for  $r_\rho$  and  $\delta_\rho$ . That is, our generic assumptions,  $r_\rho \leq 1$  and  $|\delta_\rho| \leq \pi/2$  for longitudinally polarized  $\rho$  mesons, imply  $\alpha_{\text{eff}}^{\rho\rho} > \alpha$ .

Although these assumptions about  $r_\rho$  and  $\delta_\rho$  seem reasonable [42], one may attempt to verify them experimentally, as demonstrated above for  $B \rightarrow \pi\pi$ . Current measurements of  $B(t) \rightarrow \rho^+\rho^-$  are consistent with a zero CP asymmetry [14],

$$S_{\rho\rho}^{\text{long}} = -0.19 \pm 0.35, \quad C_{\rho\rho}^{\text{long}} = -0.23 \pm 0.28, \quad |\alpha_{\text{eff}}^{\rho\rho} - \alpha| < 15^\circ \text{ (90\% CL)}. \quad (12)$$

The central values of  $S_{\rho\rho}^{\text{long}}$  and  $C_{\rho\rho}^{\text{long}}$  imply  $\alpha_{\text{eff}}^{\rho\rho} = 95.6^\circ$ . An analysis, based on expressions for  $C_{\rho\rho}^{\text{long}}$ ,  $S_{\rho\rho}^{\text{long}}$  and  $\alpha_{\text{eff}}^{\rho\rho} - \alpha$  analogous to Eqs. (4) (5) and (8), in terms  $r_\rho$ ,  $\delta_\rho$ , and  $\gamma$ , proceeds as in  $B \rightarrow \pi\pi$ . The rather low upper bound,  $|\alpha_{\text{eff}}^{\rho\rho} - \alpha| < 15^\circ$ , implies a small value for  $r_\rho$ , as can be seen in Eq. (8) where  $\alpha_{\text{eff}}^{\rho\rho} - \alpha$  vanishes in the limit  $r_\rho \rightarrow 0$ . A small value of  $r_\rho$  means a small penguin amplitude and a low sensitivity to the value of  $\delta_\rho$ .

We have verified the above statement by performing a  $\chi^2$  fit based on Eq. (12). In Fig. 1b, we show the 90% CL for the parameters  $r_\rho$  and  $\delta_\rho$ . We see that while  $r_\rho > 1$  is excluded ( $0 < r_\rho < 0.55$ ), values of  $\delta_\rho$  are permitted in the entire range  $-\pi < \delta_\rho \leq \pi$ . This situation is unlikely to change with a reduction of errors in  $S_{\rho\rho}^{\text{long}}$  and  $C_{\rho\rho}^{\text{long}}$  if  $r_\rho$  is much smaller than one as calculated in naive factorization [42].

The situation in  $B \rightarrow \rho^\pm\pi^\mp$  is somewhat more complicated than the one in  $B \rightarrow \pi^+\pi^-$  and  $B \rightarrow (\rho^+\rho^-)_{\text{long}}$  because the final states  $\rho^\pm\pi^\mp$  are not CP-eigenstates. Time-dependent decay rates for initially  $B^0$  decaying into  $\rho^\pm\pi^\mp$  are given by [43]:

$$\Gamma(B^0(t) \rightarrow \rho^\pm\pi^\mp) \propto 1 + (C \pm \Delta C) \cos \Delta mt - (S \pm \Delta S) \sin \Delta mt. \quad (13)$$

For initially  $\overline{B}^0$  decays the  $\cos \Delta mt$  and  $\sin \Delta mt$  terms have opposite signs. One defines a measurable phase  $\alpha_{\text{eff}}^{\rho\pi}$  [11], which equals  $\alpha$  in the limit of vanishing penguin amplitudes,

$$\alpha_{\text{eff}}^{\rho\pi} \equiv \frac{1}{4} \left[ \arcsin \left( \frac{S + \Delta S}{\sqrt{1 - (C + \Delta C)^2}} \right) + \arcsin \left( \frac{S - \Delta S}{\sqrt{1 - (C - \Delta C)^2}} \right) \right]. \quad (14)$$

The phase  $\alpha_{\text{eff}}^{\rho\pi}$  can be expressed in terms of parameters defining decay amplitudes. One denotes two distinct amplitudes for  $B^0 \rightarrow \rho^+\pi^-$  and  $B^0 \rightarrow \rho^-\pi^+$  by the charge of the  $\rho$  meson,

$$A(B^0 \rightarrow \rho^+\pi^-) = |t_+|(e^{i\gamma} + r_+e^{i\delta_+}), \quad (15)$$

$$A(B^0 \rightarrow \rho^-\pi^+) = |t_-|(e^{i\gamma} + r_-e^{i\delta_-}), \quad (16)$$

where  $r_{\pm} \equiv |p_{\pm}/t_{\pm}|$  are two ratios of penguin and tree amplitudes, and  $-\pi < \delta_{\pm} \leq \pi$  are the corresponding relative strong phases. Continuing the analogy with  $B \rightarrow \pi^+\pi^-$  by defining

$$\alpha_{\text{eff}}^{\rho\pi^{\pm}} \equiv \frac{1}{2} \text{Arg} \left[ e^{-2i\beta} \frac{A(\overline{B}^0 \rightarrow \rho^{\mp}\pi^{\pm})}{A(B^0 \rightarrow \rho^{\pm}\pi^{\mp})} \right], \quad (17)$$

one has

$$\alpha_{\text{eff}}^{\rho\pi} = \frac{1}{2}(\alpha_{\text{eff}}^{\rho\pi^+} + \alpha_{\text{eff}}^{\rho\pi^-}). \quad (18)$$

The two angle differences  $\alpha_{\text{eff}}^{\rho\pi^{\pm}} - \alpha$  have expressions similar to Eq. (8):

$$\alpha_{\text{eff}}^{\rho\pi^{\pm}} - \alpha = \frac{1}{2} \arctan \left[ \frac{2r_{\pm} \sin \gamma (\cos \delta_{\pm} + r_{\pm} \cos \gamma)}{1 - r_{\pm}^2 + 2r_{\pm} \cos \gamma (\cos \delta_{\pm} + r_{\pm} \cos \gamma)} \right]. \quad (19)$$

In the limit  $r_{\pm} \rightarrow 0$  one obviously obtains  $\alpha_{\text{eff}}^{\rho\pi^{\pm}} \rightarrow \alpha$ .

Flavor SU(3) relates tree and penguin amplitudes in  $B \rightarrow \rho\pi$  to corresponding contributions in  $B \rightarrow K^*\pi$  and  $B \rightarrow \rho K$ . Introducing SU(3) breaking in tree amplitudes in terms of ratios of suitable decay constants, and using measured decay rates for the above processes, the following two upper bounds were obtained at 90% CL [11]:  $|\alpha_{\text{eff}}^{\rho\pi^+} - \alpha| < 11^\circ$ ,  $|\alpha_{\text{eff}}^{\rho\pi^-} - \alpha| < 15^\circ$ . The algebraic average of these bounds provides an upper limit on  $|\alpha_{\text{eff}}^{\rho\pi} - \alpha|$ ,

$$|\alpha_{\text{eff}}^{\rho\pi} - \alpha| < \frac{1}{2} (|\alpha_{\text{eff}}^{\rho\pi^+} - \alpha| + |\alpha_{\text{eff}}^{\rho\pi^-} - \alpha|) < 13^\circ. \quad (20)$$

This bound, which may be modified to  $17^\circ$  by SU(3) breaking other than in tree amplitudes and by small annihilation amplitudes which have been neglected, does not assume knowledge of the signs of  $\alpha_{\text{eff}}^{\rho\pi^{\pm}} - \alpha$ . It becomes stronger when  $\alpha_{\text{eff}}^{\rho\pi^+} - \alpha$  and  $\alpha_{\text{eff}}^{\rho\pi^-} - \alpha$  have opposite signs, in which case they cancel each other in  $\alpha_{\text{eff}}^{\rho\pi} - \alpha$ . This possibility will be discussed now.

The signs of  $\alpha_{\text{eff}}^{\rho\pi^{\pm}} - \alpha$  depend on values of  $r_{\pm}$  and  $\delta_{\pm}$ . As we argued in the case of  $B \rightarrow \pi^+\pi^-$ , both  $\alpha_{\text{eff}}^{\rho\pi^+} - \alpha$  and  $\alpha_{\text{eff}}^{\rho\pi^-} - \alpha$  would be positive if  $r_{\pm} \leq 1$  and  $|\delta_{\pm}| \leq \pi/2$ . Arguments for  $r_{\pm} \leq 1$  are very strong. Studies based on flavor SU(3) [11, 44] and a calculation using QCD-factorization [45] obtain rather small values  $r_{\pm} \sim 0.2$ , lying very comfortably in the range  $r_{\pm} < 1$ . Both studies obtain values for  $\delta_+$  and  $\delta_-$  lying in opposite hemispheres. The output of a global SU(3) fit is [44]  $\delta_+ = (178 \pm 14)^\circ$  and  $\delta_- = (20 \pm 20)^\circ$ . Similar values are obtained in [45], where it is being argued that these values do not differ much from the naive factorization predictions:  $\delta_+ = \pi$  and  $\delta_- = 0$  [46]. Using the above values for  $r_{\pm}$  and  $\delta_{\pm}$  we see that  $\cos \delta_- + r_- \cos \gamma > 0$  while  $\cos \delta_+ + r_+ \cos \gamma < 0$ . This implies that  $0 < \alpha_{\text{eff}}^{\rho\pi^-} - \alpha < 15^\circ$  but  $-11^\circ < \alpha_{\text{eff}}^{\rho\pi^+} - \alpha < 0^\circ$ , which means that the upper bound (20) is replaced by a stronger bound,

$$-6^\circ < \alpha_{\text{eff}}^{\rho\pi} - \alpha < 8^\circ. \quad (21)$$

That is, given  $r_{\pm} \ll 1$ , and assuming that the phases  $\delta_+$  and  $\delta_-$  lie in opposite hemispheres, improves the upper bound (20) by about a factor two. The actual upper and lower bounds in (21) may be larger by about 30% because of possible

SU(3) breaking corrections in penguin amplitudes and small annihilation amplitudes which have not been taken into account.

A direct verification of our assumptions about  $\delta_{\pm}$  in  $B \rightarrow \rho^{\pm}\pi^{\mp}$  relies on interference between tree and penguin amplitudes in these processes. A possible evidence for such interference is the measured direct CP asymmetry in  $B^0 \rightarrow \rho^-\pi^+$  [35, 47, 48],

$$A_{\text{CP}}(B^0 \rightarrow \rho^-\pi^+) = -0.48 \pm 0.14 . \quad (22)$$

This asymmetry provides one equation for  $r_-$ ,  $\delta_-$  and  $\gamma$  [11] favoring negative values of  $\delta_-$ . Similarly, the direct CP asymmetry in  $B^0 \rightarrow \rho^+\pi^-$  [35, 47, 48]

$$A_{\text{CP}}(B^0 \rightarrow \rho^+\pi^-) = -0.16 \pm 0.09 , \quad (23)$$

provides an equation for  $r_+$ ,  $\delta_+$  and  $\gamma$ . Two other observables depending on these four hadronic parameters and on  $\gamma$  are  $\bar{S}$  and  $\Delta\bar{S}$ , related to  $S$  and  $\Delta S$  by a simple transformation [11]. They obtain values [47, 48]

$$\bar{S} = -0.13 \pm 0.11 , \quad \Delta\bar{S} = 0.07 \pm 0.11 \text{ (0.19)} . \quad (24)$$

The error in parentheses includes a rescaling factor of 1.7.

The dependence of  $\bar{S}$  and  $\Delta\bar{S}$  on a small relative phase  $\delta_t$  [11, 44, 45] between the two tree amplitudes  $t_+$  and  $t_-$  can be neglected. Alternatively, one may use a constraint from the phase  $\text{Arg}[A(B^0 \rightarrow \rho^+\pi^-)A^*(B^0 \rightarrow \rho^-\pi^+)]$  measured through the interference of the  $\rho^+$  and  $\rho^-$  overlapping resonances in the  $B^0 \rightarrow \pi^+\pi^-\pi^0$  Dalitz plot [47]. Another constraint on  $r_{\pm}$ ,  $\delta_{\pm}$  and  $\gamma$  is provided by the upper bound (20), where  $\alpha_{\text{eff}}^{\rho\pi^{\pm}} - \alpha$  are given in Eq. (19). This completes a system of four (or five) equations and one inequality for five parameters (or six, if we include  $\delta_t$ ). An important question is whether these constraints imply that  $\delta_+$  and  $\delta_-$  lie in opposite hemispheres, thereby leading to Eq. (21). In any case, the inclusion of these five or six relations is expected to reduce the uncertainty in  $\alpha$  below the upper bound (20).

In conclusion, we have shown that the generic assumptions,  $|\text{Tree}| \geq |\text{Penguin}|$  and  $|\text{Arg}(\text{Penguin}/\text{Tree})| \leq \pi/2$ , predict  $\alpha < \alpha_{\text{eff}}$  in  $B \rightarrow \pi^+\pi^-$  and  $B \rightarrow \rho^+\rho^-$ , thereby reducing the theoretical uncertainty in  $\alpha$  by a factor two. In the decay processes  $B \rightarrow \rho^{\pm}\pi^{\mp}$ , where two pairs of tree and penguin amplitudes occur, the assumption that the two relative phases between tree and penguin amplitudes lie in opposite hemispheres results in a suppression of about a factor two of the bound on  $|\alpha_{\text{eff}} - \alpha|$  due to cancellation between two terms. Measured CP asymmetries and  $\alpha_{\text{eff}} - \alpha$  were studied in terms of Penguin-to-Tree ratios and their relative strong phases. We presented theoretical arguments, based on QCD-factorization and on flavor SU(3), and experimental evidence (although not yet conclusive) that the various hadronic parameters do lie in ranges that allow a reduction of the error on  $\alpha$ .

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